Geometric Transformations

Standards:

Geometry: Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1: Very experimentally the properties of rotations, reflections and translations:
a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

8.G.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures describe a sequence that exhibits the congruence between them.

Mathematical Practices:

2. Reason abstractly and quantitatively.
4. Model with mathematics
5. Use appropriate tools strategically.

Learning Objectives:

Students will be able to:

Define and apply the properties of transformations to a variety of figures using both the coordinate plane and the plane in general.

Understand two figures are congruent if one can be mapped to another through a sequence of translations, rotations and reflections.

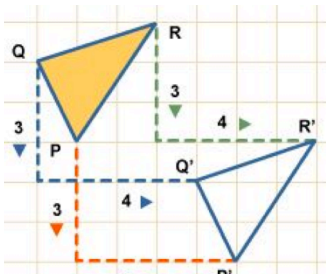
Notes:

Geometric Transformations

Definition

A *translation* is a RIGID motion that moves each point through a given distance in a given direction, line segment. A translation not only preserves distance it also preserves orientation. A composition of two translations is a translation.

Picture



Translation

In my own words:

Function Notation:

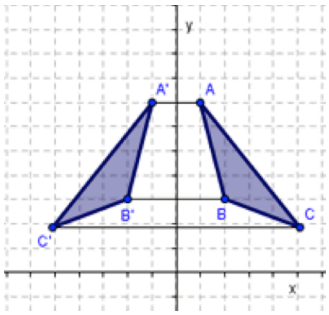
$$T_{<\text{point}>}\text{object}$$

$$T_{<-3,2>}\triangle ABC$$

Definition

A *reflection* fixes every point on a given line (axis) and maps every other point to that point such that the segment between these points is perpendicular to the axis and bisected by it. A reflection preserves distance but not orientation.

Picture



Reflection

In my own words:

Function Notation:

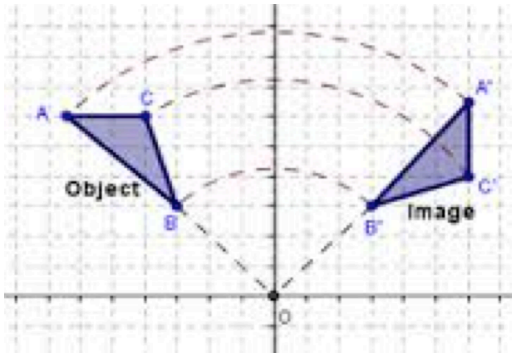
$$R_{<\text{about}>}\text{object}$$

$$R_{<linea>}\triangle ABC$$

Definition

A *rotation* is a RIGID motion that moves each point about a fixed point through a given angle. A rotation also preserves orientation as well as distance

Picture



Rotation

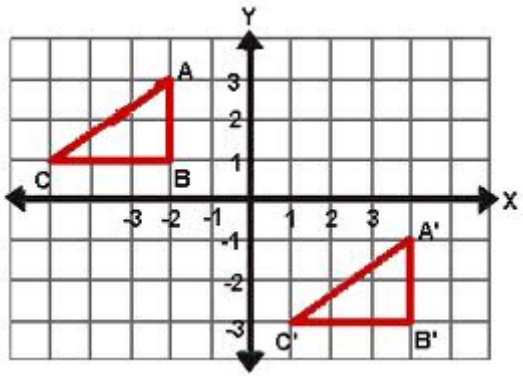
In my own words:

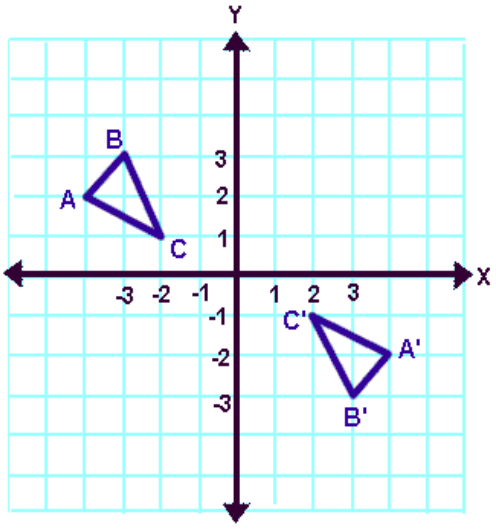
Function Notation:

$$r_{(center,direction)}\text{degrees}$$

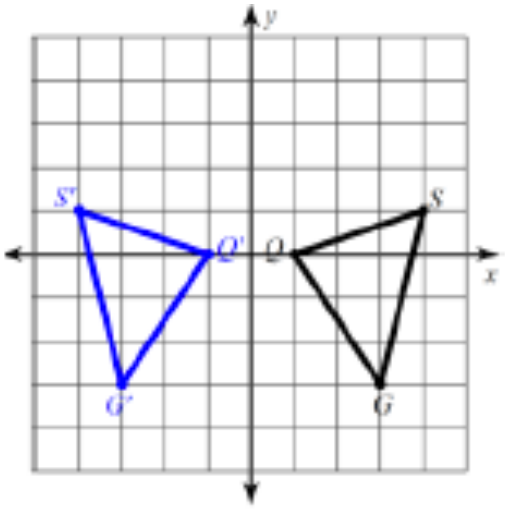
$$r_{(3,-2)cc}75^{\circ}$$

Describe fully the **single** transformation that maps triangle ABC onto triangle A'B'C'.

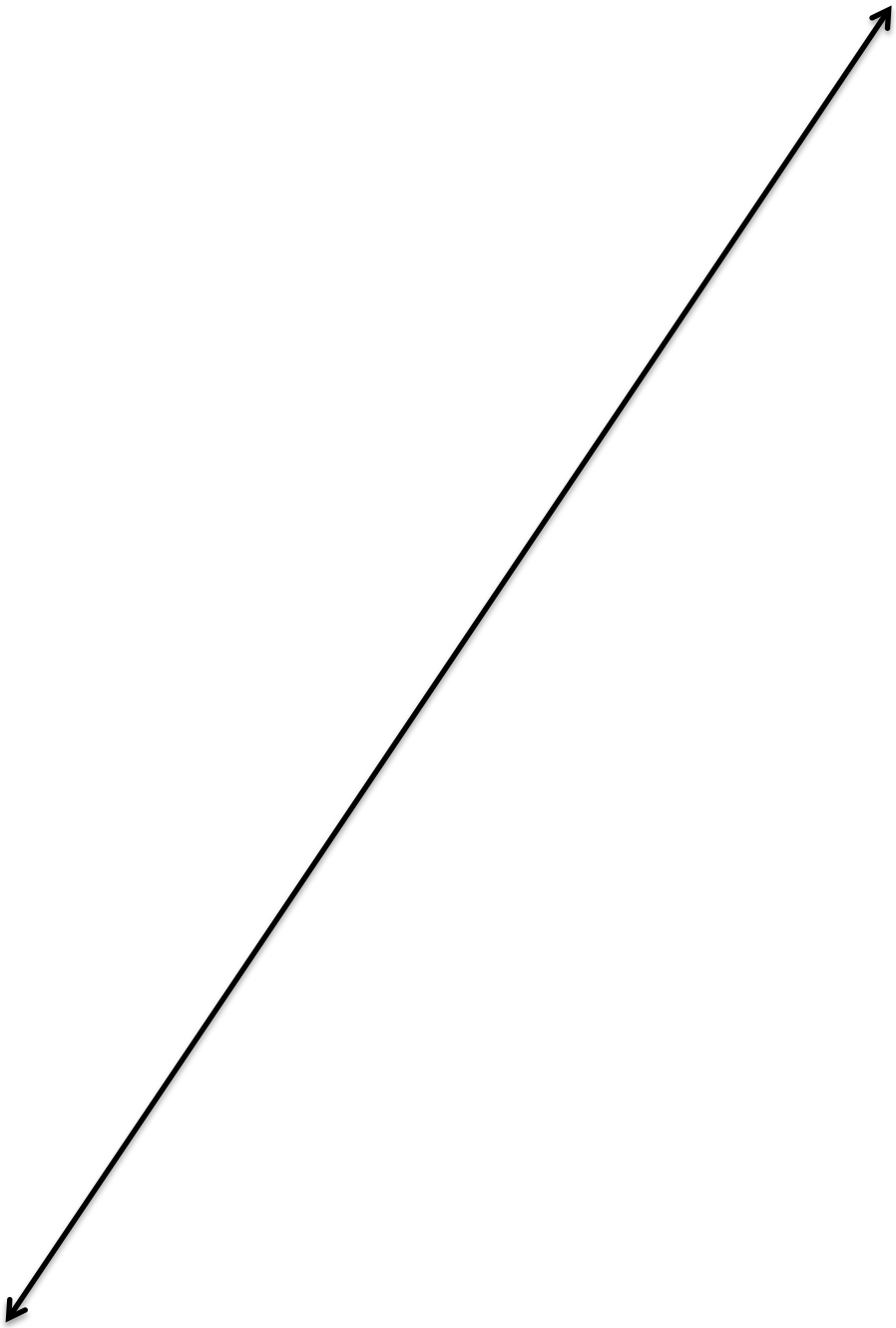




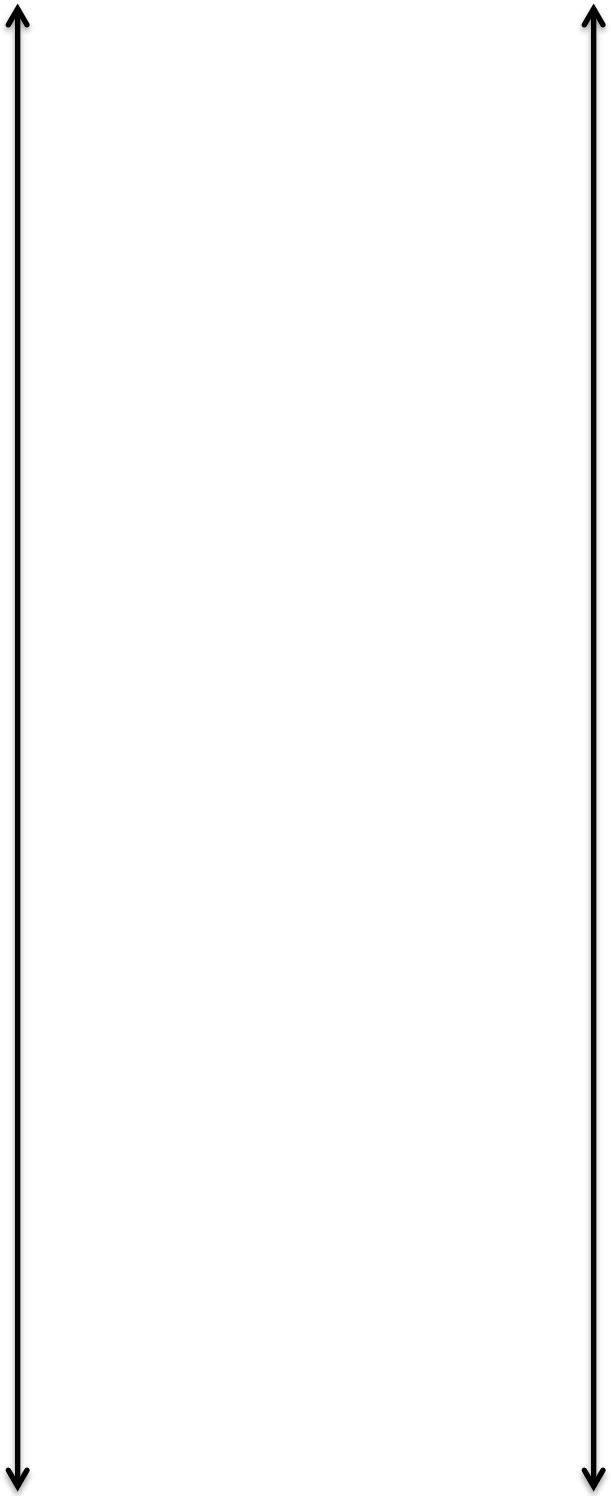
Describe fully the single transformation which maps triangle QSG onto triangle Q'S'G'.

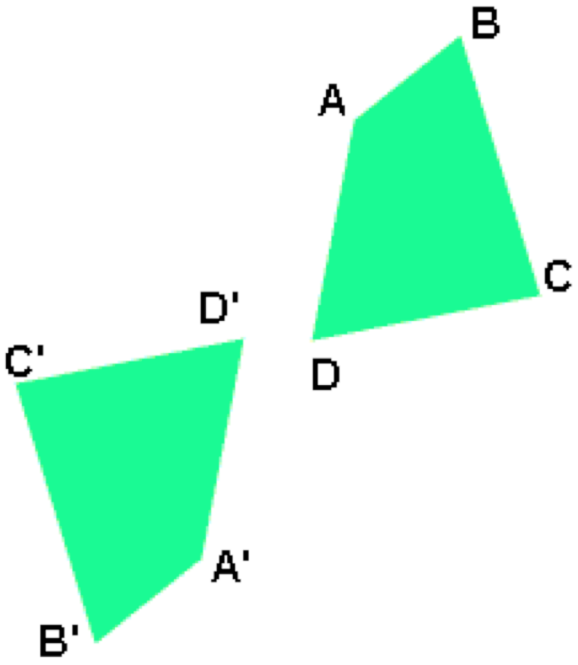


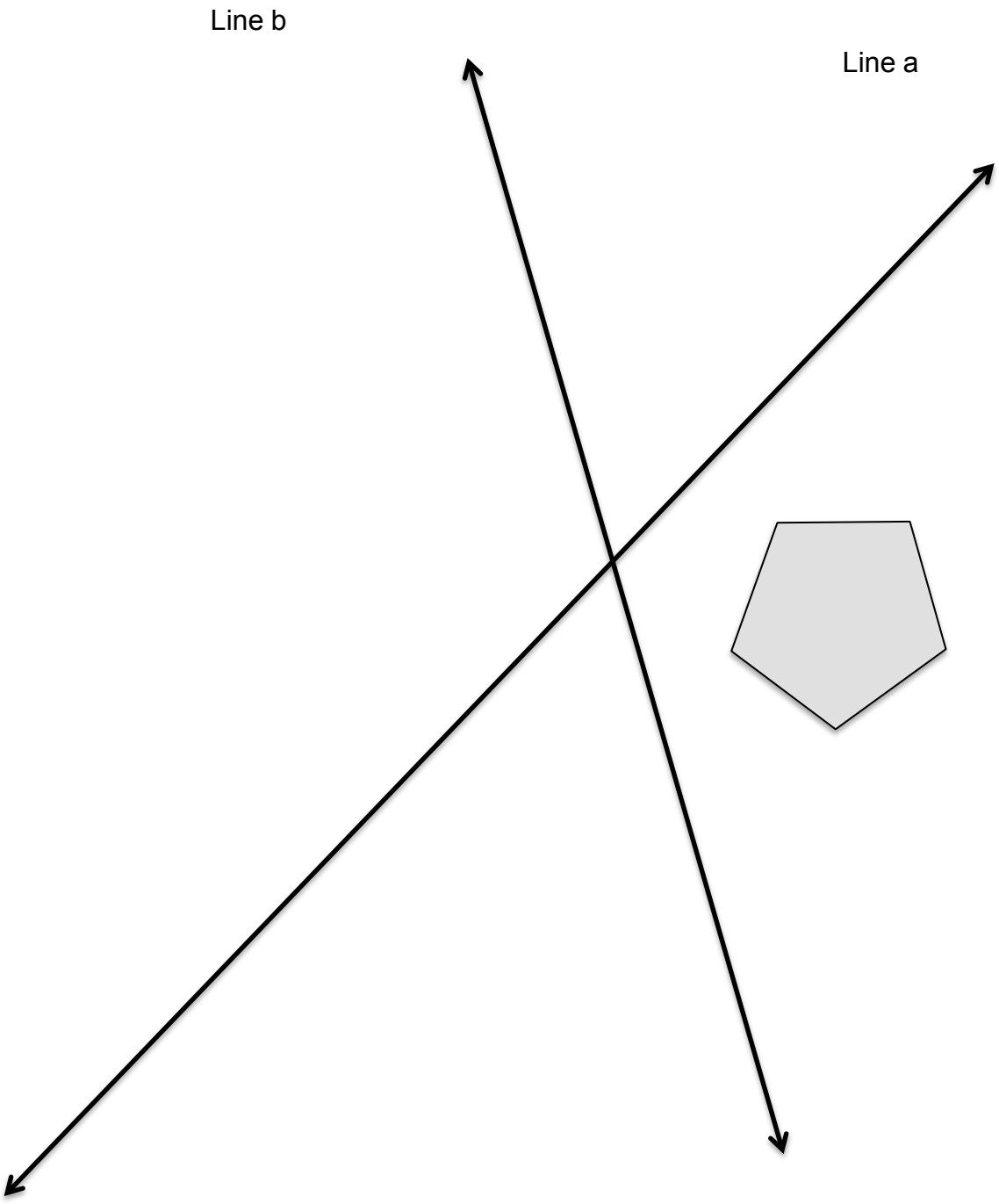
Line a

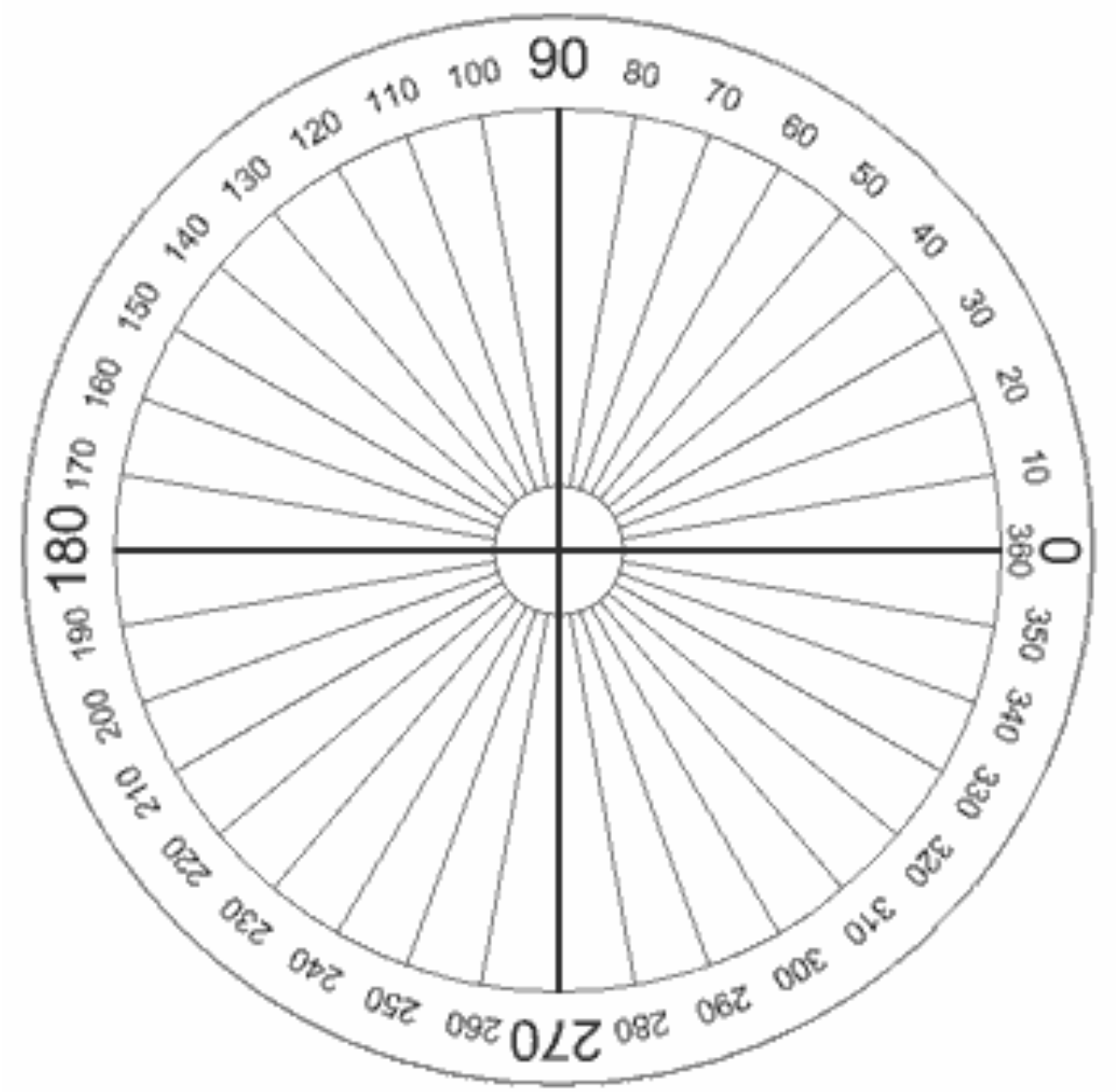


Translation:




B
A
C
D'
C'
D
A'
B'


Line b
Line a


90
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50
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10
0
360
350
340
330
320
310
300
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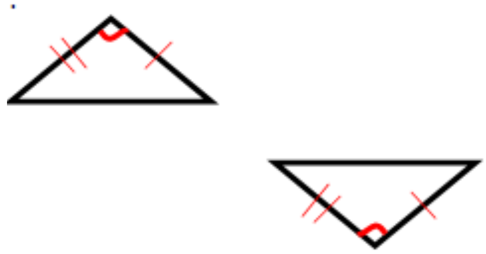
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100

Determine if the paird of triangles are congruent. If they are, state the rigid motions that would justify the triangles being congruent. Then state the congruence postulate that would show that the triangles are congruent.



Rigid Motions:

Congruence Postulate:

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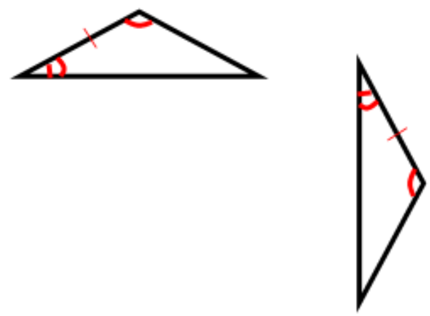

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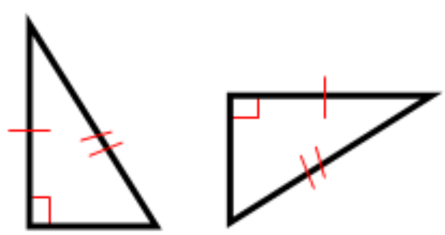
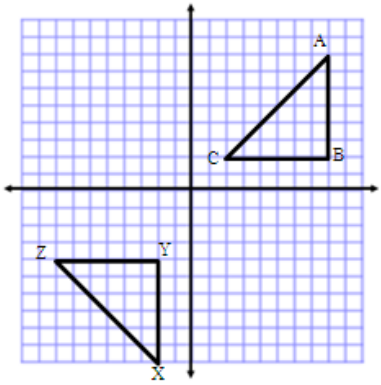


Rigid Motions:

Congruence Postulate:

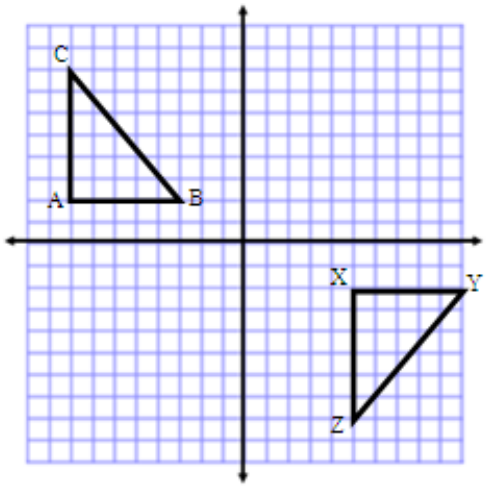
Explain why the triangle are congruent using one or more reflections, rotations, and translations. Be specific. Then describe how the triangles are congruent using congruence criteria.



Rigid Motions:

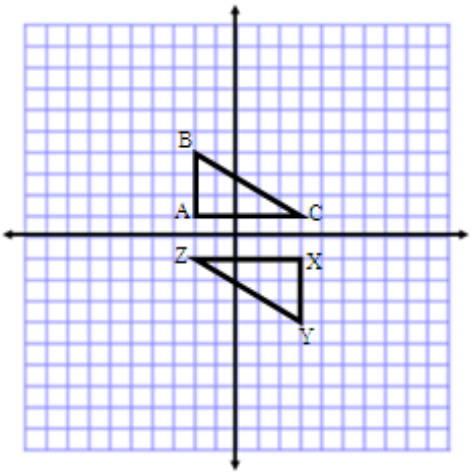
Triangle Congruence:

Explain why the triangle are congruent using one or more reflections, rotations, and translations. Be specific. Then describe how the triangles are congruent using congruence criteria



Rigid Motions:

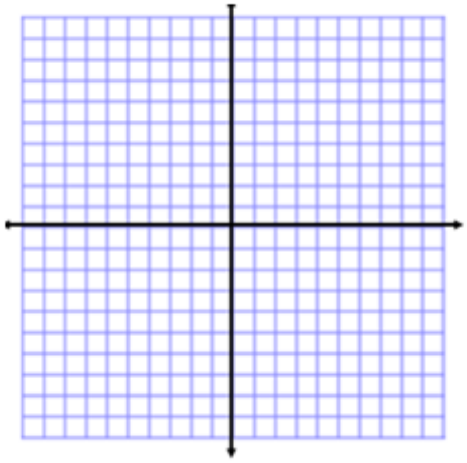
Explain why the triangle are congruent using one or more reflections, rotations, and translations. Be specific. Then describe how the triangles are congruent using congruence criteria



Rigid Motions:

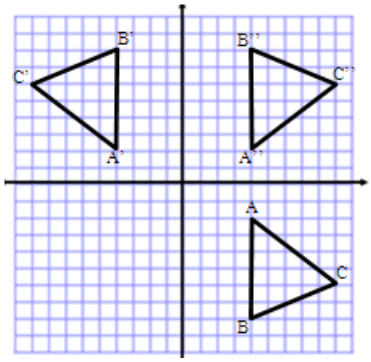
Triangle Congruence:

Given the flowing points explain <u>HOW</u> the triangles are congruent using triangle congruence criteria.
A(-7,-8), B(-7,-3), C(-1,-8), X(8,7), Y(3,7), and Z(8,1)



Triangle Congruence:

Are ΔABC and ΔA''B''C'' congruent?
If so state the transformations that were applied.
BE SPECIFIC



A' = _____ B' = _____ C' = _____

A" = _____ B" = _____ C" = _____

RULE:

Are ΔABC and ΔA''B''C'' congruent?
If so state the transformations that were applied.
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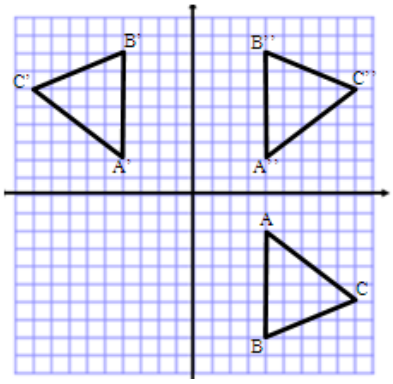
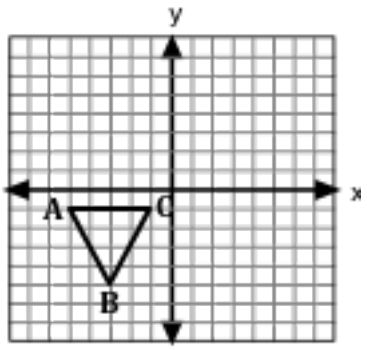
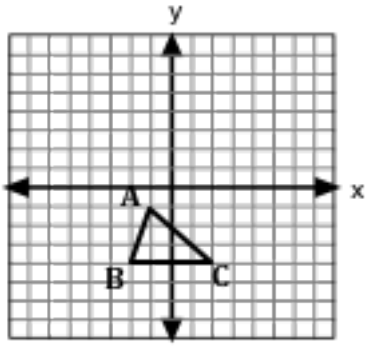
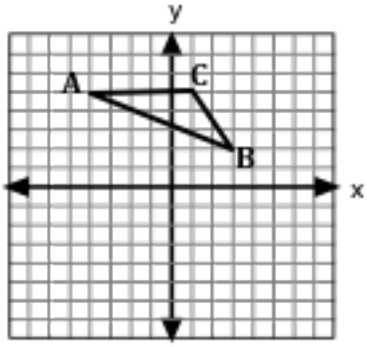


A' = _____ B' = _____ C' = _____

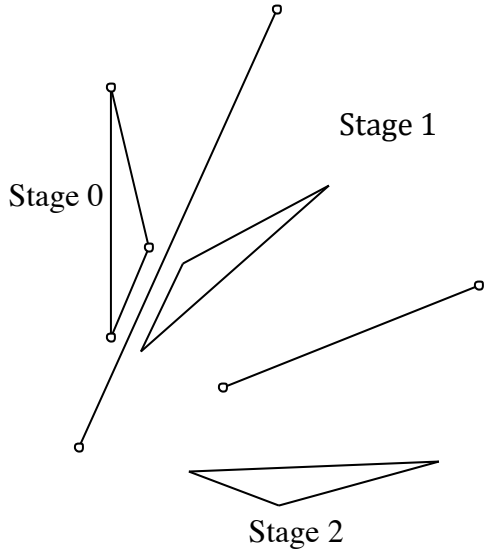
A" = _____ B" = _____ C" = _____

RULE:

	Perform the given transformations and identify the new coordinates. translate by vector ⟨-1, 6⟩, 90° clockwise  A' = _____ B' = _____ C' = _____ A" = _____ B" = _____ C" = _____ **RULE:**
Perform the given transformations and identify the new coordinates. translate by vector ⟨2, -2 ⟩, 90° counterclockwise  A' = _____ B' = _____ C' = _____ A" = _____ B" = _____ C" = _____ **RULE:**	Perform the given transformations and identify the new coordinates. translate by vector ⟨-2, 0⟩, reflect across y-axis  A' = _____ B' = _____ C' = _____ A" = _____ B" = _____ C" = _____ **RULE:**

Quiz A Name:_____

Use the following diagram to answer the questions below. You may use a protractor and ruler as needed.



1. Describe the transformation from the pre-image at stage 0 to the image at stage 1.

1. Describe the transformation from the image at stage 1 to the image at stage 2.

2. Describe a single transformation of the plane that takes the pre-image to the the image at stage 2.

Quiz B **Name:**_____

1) A rigid motion J of the plane takes a point A as input and gives C as output, i.e., $J(A) = C$. Similarly, $J(B) = D$ for input point B and output point D.

Jerry claims that knowing nothing else about J, we can be sure that $\overline{AC} \cong \overline{BD}$ because rigid motions preserve distance.

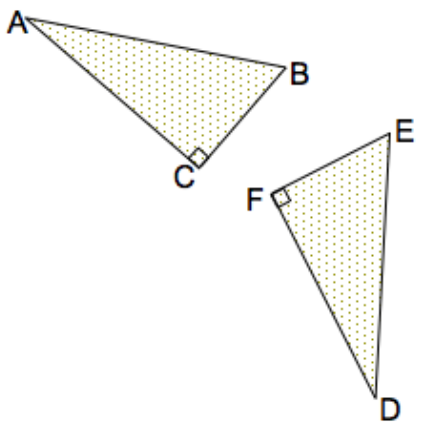
a. Show that Jerry's claim is incorrect by giving a counterexample (hint: a counterexample would be a specific rigid motion and four points A, B, C, and D in the plane such that the motion takes A to C and B to D, yet $\overline{AC} \ncong \overline{BD}$).

b. There is a type of rigid motion for which Jerry's claim is always true. Which type below is it?

Rotation Reflection Translation

c. Suppose Jerry claimed that $\overline{AB} \cong \overline{CD}$. Would this be true for any rigid motion that satisfies the conditions described in the first paragraph? Why or why not?

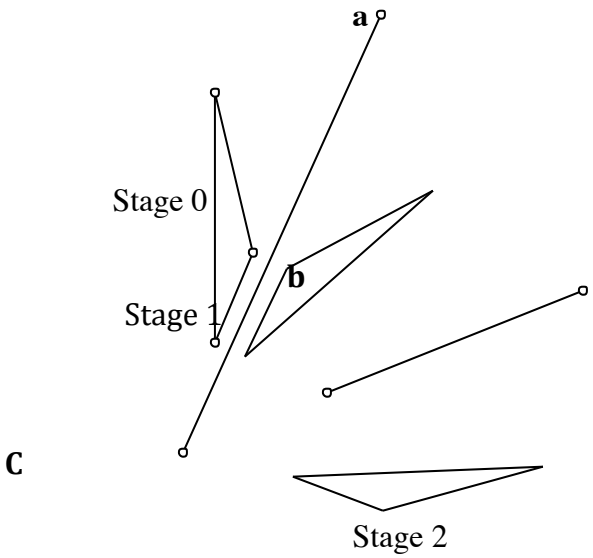
2) For the triangles ΔABC and ΔDEF in the figure below $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$.



a. Using the given information, what criteria for triangle congruence (ASA, SAS, SSS) implies that $\Delta ABC \cong \Delta DE$?

b. Describe a sequence of rigid transformations that shows $\Delta ABC \cong \Delta DEF$.

Key to Quiz A



1. reflection across line a (see above)
2. reflection across line b (see above)
3. 80 – 90 degree clockwise rotation about point C (see above)

Key to Quiz B

1	**a – c** **G-CO.2**	"Translation" is circled for part (b), but no further response is correct or shows clear understanding of the application of rigid motions.	The response includes a counterexample provided in part (a) OR an idea is presented to prove that $\overline{AB} \cong \overline{CD}$ in part (c), whichever is presented is less than perfectly clear in stating the solutions; "translation" is circled for part (b).	The response includes a counterexample in part (a) AND an idea is presented to prove that $\overline{AB} \cong \overline{CD}$ in part (c), but both are less than perfectly clear in stating the solutions; "translation" is circled for part (b).	The response includes is a correctly reasoned counterexample in part (a), "translation" is circled for part (b), AND a justified claim that $\overline{AB} \cong \overline{CD}$ for any rigid motion in part (c).

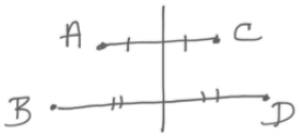
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
2	**a – b** **G-CO.7** **G-CO.8**	A response shows little or no evidence of understanding for part (a) or (b).	A response shows the correct triangle congruence criteria listed in part (a) and lists only one rigid transformation for part (b).	A response shows the correct triangle congruence criteria listed in part (a) and lists only two rigid transformations for part (b).	A response shows the correct triangle congruence criteria listed in part (a) any order of translation, rotation, and reflection in part (b).

Sample Student Solution

2. A rigid motion, J, of the plane takes a point, A, as input and gives C as output, i.e., JA= C. Similarly, JB= D for input point B and output point D.

   Jerry claims that knowing nothing else about J, we can be sure that $\overline{AC} \cong \overline{BD}$ because rigid motions preserve distance.

   a. Show that Jerry's claim is incorrect by giving a counterexample (hint: a counterexample would be a specific rigid motion and four points A, B, C, and D in the plane such that the motion takes A to C and B to D, yet $AC \ncong BD$).



Here, J is a reflection across a vertical line. The distance from A to the line is different from the distance from B to the line. Therefore, the distance from A to its image (C) is different from the distance from B to its image (D).

   b. There is a type of rigid motion for which Jerry's claim is always true. Which type below is it?

Rotation Reflection (Translation)

   c. Suppose Jerry claimed that $AB \cong CD$. Would this be true for any rigid motion that satisfies the conditions described in the first paragraph? Why or why not?

Yes, because rigid motions always preserve distance.